Comparing the Traditional Oscillator Amplitude Control Loop (with ‘zero’) and Li-Tsividis’s Direct-Q Control Loop

Personal Report

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I. Introduction

The objective under consideration is to control the amplitude of an oscillator (in the *GHz* range) to sufficiently small values so that any nonlinear effect becomes negligible. This will allow the use of such oscillator in tuning of filters made of similar structures.

For a generic second order oscillator described by

$$\frac{d^2 v_o(t)}{dt} + b(t)\frac{dv_o(t)}{dt} + \omega_o^2 v_o(t) = 0, \tag{1}$$

if $b(t) = \omega_o/Q(t) = 0$, then $v_o(t) = A_o\cos(\omega t)$, where A_o is the constant oscillator amplitude, set (arbitrarily) by initial conditions. An automatic amplitude control loop can be used to adjust $b(t)$ exactly at ‘0’ and make the final oscillator amplitude be set at a specified reference voltage V_{REF}. Fig. 1 illustrates this. It is known that, if $b(t)$ stays sufficiently close to zero so that the oscillator frequency can be considered constant, then [1]-[3]

$$A(t) \approx A(t_o)\exp\left(-\frac{1}{2}\int_{t_o}^{t} b(t)dt\right) \tag{2}$$

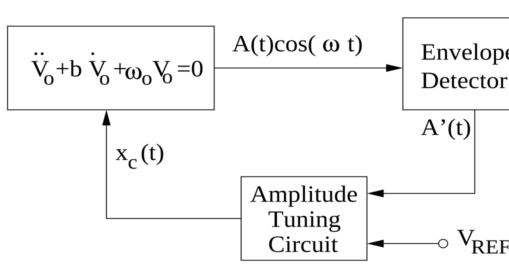

Fig. 1: Generic Oscillator Amplitude Control Structure for adjusting Amplitude to a Reference

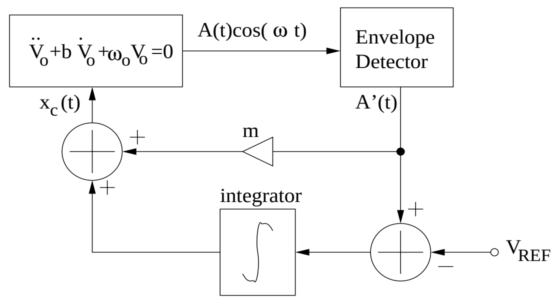

Fig. 2: Classic Oscillator Amplitude Control Loop based on Integrating the Error between a Reference Amplitude and the Envelope Detector Output, and adding a 'zero' for Stability

If $b(t)$ stays sufficiently close to '0' (for example, it suffers small perturbations above and below '0') then the integral also stays close to '0'. This allows us to use Taylor expansion on the exponential,

$$A(t) \approx -\frac{A(t_o)}{2}\int_{t_o}^{t} b(t)dt \tag{3}$$

In the *s*-domain, eq. (3) becomes

$$A(s) \approx -\frac{A_o}{2}\frac{b(s)}{s} \tag{4}$$

Term $b(s)$ is physically controlled by external signal $x_c(s)$. Let us assume both are related by

$$b(s) = \alpha x_c(s) \tag{5}$$

The envelope detector output may generally be modelled with the following imperfections

- attenuation ρ $(0 < \rho < 1)$
- level shift
- delay τ_{env}

In the s-domain we ignore the level shift because it is a DC property. Consequently, at the envelope detector output we will have

$$A'(s) = \rho A(s)(1 - s\tau_{env}) \tag{6}$$

Eqs. (4)-(6) are valid for any implementation of the "Amplitude Tuning Circuit" in Fig. 1. In what follows we will compare two alternative implementations for this block [1], [4]. In the appendixes alternative analyses are provided based on modeling the delays with a 'pole' instead of a 'zero'.

II. Classic Amplitude Error-Integration Based AGC with 'zero'

The circuit diagram in Fig. 2 illustrates the classic oscillator amplitude control loop based on integrating the error resulting from comparing the envelope detector output with a reference. A direct feed forward path is added around the integrator for stability [5][1]. For this circuit

$$x_c(s) = mA'(s) + \frac{A'(s) - V_{REF}(s)}{s\tau_{agc}} \qquad (7)$$

Solving eqs. (4)-(7) results in

$$\frac{A(s)}{V_{REF}(s)} = \frac{1/\rho}{s^2 k_1 + s k_2 + 1}$$
$$k_1 = \left(\frac{2}{\alpha A_o \rho} - m\tau_{env}\right)\tau_{agc} \qquad (8)$$
$$k_2 = m\tau_{agc} - \tau_{env}$$

Stability is guaranteed for $k_1 > 0$ and $k_2 > 0$. Parameters m and τ_{agc} have to be chosen so that k_1 and k_2 are positive for worst case α, ρ, A_o and τ_{env}. Note that depending on the circuit chosen for the envelope detector, τ_{env} and ρ may be functions of A. In summary, the stability conditions are

$$m > \frac{\tau_{env}}{\tau_{agc}}$$
$$m < \frac{2}{\alpha A_o \rho \tau_{env}} \qquad (9)$$

The top inequality is amplitude independent, while the bottom one needs to be adjusted for maximum oscillator amplitude ($A_o = A_{max}$). Making $m = 0$ yields an unstable control loop. Therefore, the 'zero' (i.e., the presence of amplifier 'm') is required for stability.

In Appendix 1 an alternative analysis is provided based on modelling the envelope detector with a 'pole' instead of a 'zero'. The resulting stability condition is only the top inequality of eq. (9), which is amplitude independent.

Assuming a stabilized control loop, the steady state for the amplitude A (i.e, after the transient caused by a step in V_{REF}, for example) is obtained from eq. (8) by taking $s = 0$,

$$A = \frac{1}{\rho} V_{REF} \qquad (10)$$

where ρ was the attenuation of the envelope detector. In Appendix 2 further considerations are made for the result in eq. (8), in order to provide hints on how to optimize the loop parameters for optimum transient responses.

1. The authors in [5] did not model the envelope detector delay. Consequently, if there is no 'zero' (m=0) the resulting poles of the amplitude control system are exactly on the imaginary axis. However, they added the 'zero' to move them into the left half plane and assure that no higher order effect would move them into the right half plane. In [1]-[3] the envelope detector delay is modelled and therefore the use of the 'zero' is mandatory to obtain a stable amplitude control loop.

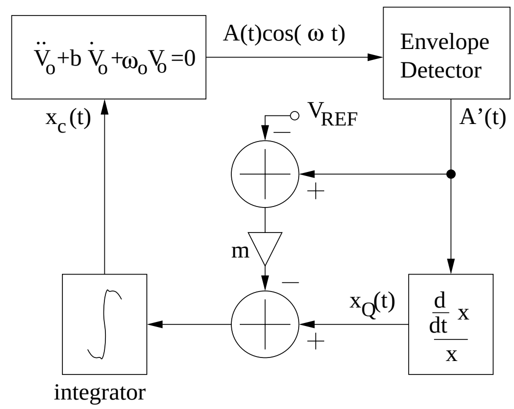

Fig. 3: Li-Tsividis's Direct-Q Tuning Loop for Oscillator Amplitude Control

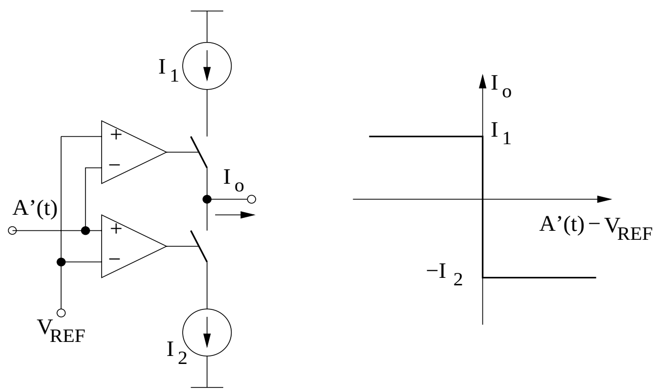

Fig. 4: Implementation of the '*m*' block of Fig. 3

III. Direct-Q Amplitude Tuning Loop

This control loop [4] is based on the observation that a measure of the oscillator $Q(t)$ can be obtained from the envelope. Taking time derivatives in eq. (2) yields,

$$\dot{A}(t) = -\frac{1}{2}b(t)A(t) \tag{11}$$

Consequently,

$$\frac{\dot{A}(t)}{A(t)} = -\frac{1}{2}b(t) = -\frac{1}{2}\frac{\omega_o}{Q(t)} \tag{12}$$

Therefore, a circuit is built that takes the envelope detector output $A'(t)$ and performs the operation in eq. (12) to extract (a delayed version of) $b(t)$. The block diagram can be drawn as shown in Fig. 3. The '*m*' block in Fig. 3 was implemented by Li and Tsividis using the circuit shown in Fig. 4. This implies an infinite slope. However, let us consider a finite *m* for the following analysis,

$$I_o(s) = -m(A'(s) - V_{REF}(s)) \tag{13}$$

which would mean we can implement the '*m*' block with a linear amplifier as well. This way, the analysis is valid for both cases.

The *b* (or *Q*) extracting circuit is not instantaneous (at least, it includes the envelope detector delay). Let us model it as

$$x_Q(s) = -kb(s)(1 - s\tau_Q) \tag{14}$$

where $\tau_Q \ge \tau_{env}$. Consequently, the gain control loop in Fig. 3 can be described by

$$\begin{aligned} A'(s) &= \rho A(s)(1 - s\tau_{env}) \\ x_Q(s) &= -kb(s)(1 - s\tau_Q) \\ I_o &= -m(A'(s) - V_{REF}(s)) \\ x_c(s) &= \frac{x_Q(s) - I_o(s)}{s\tau_{agc}} \\ b(s) &= \alpha x_c(s) \\ A(s) &= -\frac{A_o}{2}\frac{b(s)}{s} \end{aligned} \tag{15}$$

Solving this set of equations results in

$$\frac{A}{V_{ref}} = \frac{1/\rho}{s^2\frac{2}{m\rho A_o}\left(\frac{\tau_{agc}}{\alpha} - k\tau_Q\right) + s\left(\frac{2k}{m\rho A_o} - \tau_{env}\right) + 1} \tag{16}$$

For stability, the s^2 and *s* coefficients must be positive,

$$\begin{aligned} &\tau_{agc} > \alpha k \tau_Q \\ &m < \frac{2k}{\rho A_o \tau_{env}} \end{aligned} \tag{17}$$

Appendix three shows an alternative analysis in which the envelope detector and the *Q* extracting circuit are modeled with 'poles' instead of 'zeros'.

From eq. (17), setting $m = \infty$ would yield an unstable amplitude control loop. This is what happens in the results shown by Li and Tsividis (see Fig. 10(a) of [4]). However, the envelope oscillates with a stable amplitude. This amplitude (let's call it v_m) can be predicted from eq. (16) by treating the '*m*' element with the "Describing Function" [6]. In this case, the output of the comparator is approximated by its first Fourier expansion term, while assuming a sinusoid at the comparator input v_{in}. Under these assumptions the comparator behaves as a linear amplifier,

$$I_o \approx m_d v_{in} \tag{18}$$

where m_d is a nonlinear decreasing function of the input signal (v_{in}) peak amplitude (v_m) [6]

$$m_d(v_m) = \frac{4I_1}{\pi v_m} \tag{19}$$

where $I_1 = I_2$ has been assumed.

The oscillating amplitude of the envelope $A(t)$ (or $A(s)$), v_m, will be constant if in eq. (16) the s coefficient is zero,

$$m_d = \frac{4I_1}{\pi v_m} = \frac{2k}{A_o \rho \tau_{env}} \qquad \Rightarrow \qquad \frac{v_m}{A_o} = \frac{2I_1 \rho \tau_{env}}{k\pi} \tag{20}$$

In practice one would like to minimize v_m/A_o, by minimizing the right hand side ratio of eq. (20), i.e. by minimizing I_1. However, note that the speed of the control loop is directly proportional to I_1. Consequently, a trade off exists between ripple and speed.

IV. Appendix 1

The classic amplitude control loop of Fig. 2 can be analyzed as well by considering the envelope detector modelled by a ‘pole’ instead of a ‘zero’. In that case

$$A'(s) = \frac{\rho A(s)}{1 + s\tau_{env}} \tag{21}$$

Solving eqs. (4)-(7), but using eq. (21) instead of eq. eq. (6), results in

$$\begin{aligned} \frac{A(s)}{V_{REF}(s)} &= \frac{(1/\rho)(1 + s\tau_{env})}{as^3 + bs^2 + cs + 1} \\ a &= \frac{2\tau_{agc}\tau_{env}}{\alpha\rho A_o} \\ b &= \frac{2\tau_{agc}}{\alpha\rho A_o} \\ c &= m\tau_{agc} \end{aligned} \tag{22}$$

Using Routh criterion for stability analysis of this three pole systems results in the following stability condition ($bc > a$),

$$m > \frac{\tau_{env}}{\tau_{agc}} \tag{23}$$

Note that making $m = 0$ will inevitably yield instability. Also note, that if τ_{env} can be considered, in first approximation, independent of signal amplitude A_o then eq. (23) is a stability condition independent of oscillator amplitude.

V. Appendix 2

For the classic amplitude control loop of Fig. 2, in case the peak detector is modeled in the frequency domain by a delay (approximated by a ‘zero’)

$$A'(s) = \rho A(s)(1 - s\tau_{env}) \qquad (24)$$

the speed of the amplitude control loop can be optimized by adjusting the poles of eq. (8)

$$s_o = -\frac{k_2}{2k_1} \pm j\frac{\sqrt{4k_1 - k_2^2}}{2k_1} \qquad (25)$$

where we are assuming

$$4k_1 - k_2^2 \geq 0 \qquad \Leftrightarrow \qquad \frac{8\tau_{agc}}{\alpha\rho A_o} \geq (m\tau_{agc} + \tau_{env})^2 \qquad (26)$$

To maximize speed we need to make the real part of s_o as large as possible

$$\frac{k_2}{2k_1} = \frac{1}{2}\frac{m\tau_{agc} - \tau_{env}}{\frac{2\tau_{agc}}{\alpha\rho A_o} - m\tau_{agc}\tau_{env}} \qquad (27)$$

For example, we can make the denominator zero (or $+\varepsilon$) for the maximum required amplitude A_{max}. This would yield

$$m = \frac{2}{\alpha\rho\tau_{env}A_{max}} \qquad (28)$$

Under this constraint eq. (27) becomes

$$\frac{k_2}{2k_1} = \frac{1}{2}\frac{\tau_{env}^{-1} - \frac{\alpha\rho}{2}A_{max}\tau_{env}\tau_{agc}^{-1}}{\frac{A_{max}}{A_o} - 1} \qquad (29)$$

Maximizing the numerator implies the condition

$$\tau_{agc} \gg \frac{\alpha\rho}{2}A_{max}\tau_{env}^2 \qquad (30)$$

Which will make the final pole real part equal to

$$Real(s_o) = -\frac{k_2}{2k_1} \approx -\frac{1}{2}\frac{A_o}{A_{max}}\frac{1}{\tau_{env}} \qquad (31)$$

For critically damped transient response one can further adjust

$$\xi = \frac{1}{2Q} = \frac{k_2}{2\sqrt{k_1}} = \frac{1}{\sqrt{2}} \tag{32}$$

which means that

$$\tau_{agc} = \alpha\rho A_{max}\tau_{env}^2\left\{\left(\frac{3}{8}+\frac{A_{max}}{2A_o}\right)+\sqrt{\left(\frac{3}{8}+\frac{A_{max}}{2A_o}\right)^2-\frac{1}{4}}\right\} \tag{33}$$

If this is adjusted for $A_{min} \ll A_{max}$, then

$$\tau_{agc} \approx \frac{\alpha\rho}{2}\tau_{env}^2\frac{A_{max}^2}{A_{min}} \tag{34}$$

VI. Appendix 3

Here we present an alternative analysis of that in Section III, based on modeling the delays in the envelope detector and Q extractor by ‘poles’ instead of ‘zeros’. In this case, the amplitude control loop in Fig. 3 is described by

$$\begin{aligned}
A'(s) &= \frac{\rho A(s)}{1+s\tau_{env}}\\
x_Q(s) &= \frac{-kb(s)}{1+s\tau_Q}\\
I_o &= -m(A'(s)-V_{REF}(s))\\
x_c(s) &= \frac{x_Q(s)-I_o(s)}{s\tau_{agc}}\\
b(s) &= \alpha x_c(s)\\
A(s) &= -\frac{A_o}{2}\frac{b(s)}{s}
\end{aligned} \tag{35}$$

Solving this set of equations results in

$$\begin{aligned}
\frac{A(s)}{V_{REF}(s)} &= \frac{m\alpha A_o(1+s\tau_{env})(1+s\tau_{agc})}{as^4+bs^3+cs^2+ds+e}\\
a &= 2\tau_{agc}\tau_Q\tau_{env}\\
b &= 2\tau_{agc}(\tau_Q+\tau_{env})\\
c &= 2\tau_{agc}+2k\alpha\tau_{env}\\
d &= 2k\alpha+m\rho\alpha A_o\tau_Q\\
e &= m\rho\alpha A_o
\end{aligned} \tag{36}$$

Using Routh stability criterion on eq. (36) yields

$$\begin{aligned} bc - ad &> 0 \\ (bc - ad)d - b^2 e &> 0 \end{aligned} \tag{37}$$

The first inequality, for example, results in

$$m < 2\frac{\tau_{agc}\tau_Q + \tau_{env}(\tau_{agc} + k\alpha\tau_{env})}{\alpha\rho A_o \tau_Q^2 \tau_{env}} \tag{38}$$

Consequently, '*m*' has an upper bound for stability requirements.

VII. References

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